Name of College :- S.S. College, J. Bad

Deptt — Mathematics

Topic — Problem based on Direction Cosine (Analytical Geometry of 3-D)

Class . B.Sc I (Sub)

Date - 29-07-2020

Time — 1.00 P.M to 1.45 P.M

By - Dr Shree Nath Sharma

Problem :→ Show that

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Solution → Since $l = \cos\alpha$

$m = \cos\beta$

$n = \cos\gamma$

Also $l^2 + m^2 + n^2 = 1$

i.e $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

2. Show that the three lines from O with direction cosines

(l_1, m_1, n_1) (l_2, m_2, n_2) (l_3, m_3, n_3)

are coplanar if

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

Solution :→ Let (l, m, n) be the direction cosines of a line which is normal to the plane containing three given lines.

Hence $ll_1 + mm_1 + nn_1 = 0$ —— (I)

$ll_2 + mm_2 + nn_2 = 0$ —— (II)

$ll_3 + mm_3 + nn_3 = 0$ —— (III)

Eliminating (l, m, n) from (I) (II) and (III)

We get

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

8. Show that the line joining the points $(1,2,3)$, $(-1,-2,-3)$ is perpendicular to the line joining the points $(-2,1,5)$, $(3,3,2)$

[Direction ratios of a st. line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2)]

are $x_2 - x_1 = a$

$y_2 - y_1 = b$

$z_2 - z_1 = c$

Also two lines are perpendicular

if $aa_1 + bb_1 + cc_1 = 0$ where (a,b,c) and (a_1, b_1, c_1) are direction ratios

or $ll_1 + mm_1 + nn_1 = 0$ (l, m, n) and (l_1, m_1, n_1) are direction cosine.

If a, b, c are direction ratios of the line passing through the points $(1,2,3)$ and $(-1,-2,-3)$

then $a = -1-1 = -2$ $b = -2-2 = -4$ $c = -3-3 = -6$

Also (a_1, b_1, c_1) are direction ratios of the line passing through $(-2,1,5)$ and $(3,3,2)$

then $a_1 = 3+2 = 5$ $b_1 = 3-1 = 2$ $c_1 = 2-5 = -3$

Now $aa_1 + bb_1 + cc_1$

$= (-2)(5) + (-4)(2) + (-6)(-3)$

$= -10 - 8 + 18 = 0$

thus given two lines are mutual perpendicular.

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Problem :→ Show that st. lines whose direction cosines are given by

$$2l + 2m - n = 0$$
$$mn + nl + lm = 0$$ are at right-angles

Solution → By the question

$$2l + 2m - n = 0 \quad \text{———} \; (1)$$

$\Rightarrow n = (m+l)2$

Also $mn + nl + lm = 0$

$2m(m+l) + 2(l+m)l + lm = 0$ put $n = 2(m+l)$

$\Rightarrow 2m^2 + 2ml + 2l^2 + 2ml + lm = 0$

$\Rightarrow 2l^2 + 5lm + 2m^2 = 0$

$\Rightarrow 2l^2 + 4lm + lm + 2m^2 = 0$

$\Rightarrow 2l^2 + 5lm + 2m^2 = 0$

$\Rightarrow (2l+m)(l+2m) = 0$

If $2l + m = 0$

$\Rightarrow 2l + 2m = m$

$\Rightarrow n = m$

∴ From (1)

$$\frac{l}{-1} = \frac{m}{2} = \frac{n}{2}$$

⇒ Direction cosines are proportional to

$-1, 2, 2$

∴ Actual Direction cosines are

$-\frac{1}{3}, 2/3, 2/3$

If $l + 2m = 0$

$2l + im = l$

$\Rightarrow l = n$

$\therefore \frac{l}{2} = \frac{m}{-1} = \frac{n}{2}$

the Direction cosines are proportional to

$2, -1, 2$

$\therefore$ Actual direction cosines of this line are

$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$.

If θ be the angle between these two lines

then $\cos\theta = \left(-\frac{1}{3}\right)(2/3) + \left(\frac{2}{3}\right)(-1/3) + \frac{2}{3} \times \frac{2}{3}$

$= -\frac{2}{9} - \frac{2}{9} + \frac{4}{9} = 0$

$\therefore$ lines are at right angles

Problem

Find the angle between the lines whose direction cosines are given by the Equations

$l + m + n = 0$

and $2lm + 2ln - mn = 0$

Solution :-> We have $l + m + n = 0$

$\Rightarrow n = -(l+m)$ ——— (1)

Also $2lm + 2ln - mn = 0$

$2lm + 2l[-(l+m)] - m[-(l+m)] = 0$

$\Rightarrow 2lm - 2l^2 - 2lm + ml + m^2 = 0$

$\Rightarrow -2l^2 + lm + m^2 = 0$

$\Rightarrow m^2 + lm - 2l^2 = 0$

$\Rightarrow m^2 + 2lm - lm - 2l^2 = 0$

$\Rightarrow m(m+2l) - l(m+2l) = 0$

$\Rightarrow (m-l)(m+2l) = 0$

$\Rightarrow$ Either $m - l = 0$ or $m + 2l = 0$

When $m = l$

then from (1) $n = -2l$

$\therefore \frac{l}{1} = \frac{m}{1} = \frac{n}{-2}$

$\Rightarrow$ the Direction ratio of this line is $1, 1, -2$.

$\therefore$ Actual direction cosines are

$\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}$

Again when $m + 2l = 0$

$\Rightarrow m = -2l$

then from (1)

$n = l$.

$\therefore \frac{l}{1} = \frac{m}{-2} = \frac{n}{1}$

The Direction ratio of this line are $1, -2, 1$

$\therefore$ Actual Direction cosine of this lines are $\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

If θ be the angle between these two lines

then $\cos\theta = \pm(l_1 l_2 + m_1 m_2 + n_1 n_2)$

$= \pm\left(\frac{1}{\sqrt{6}}\cdot\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}\left(-\frac{2}{\sqrt{6}}\right) + \left(-\frac{2}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right)\right)$

$= \pm\left(\frac{1}{6} - \frac{2}{6} - \frac{2}{6}\right)$

$= \mp \frac{3}{6} = \mp\frac{1}{2}$

this the acute angle between these two lines is 60°